Mark Scheme (Results)

Summer 2008

GCE

GCE Mathematics (6676/01)

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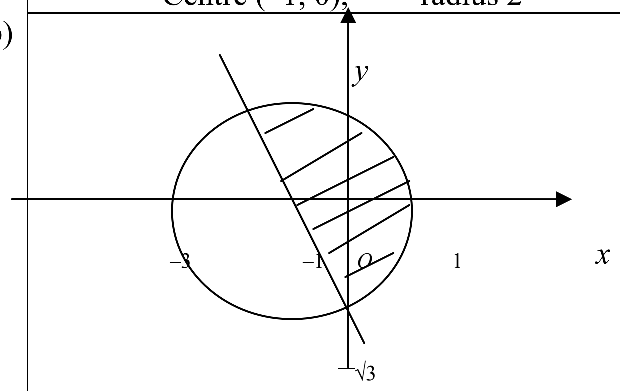
6676 Further Pure FP3
Mark Scheme

Question Number	Scheme	Marks
1.	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 = 0 + \cos 0.6 \quad (= 0.825335\ldots)$ May be implicit	B1
	$y_1 \approx 0.05\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 + y_0 \quad (= 0.05 \times 0.825335\ldots + 0.6)$	M1
	$y_1 \approx 0.641266\ldots$ $= 0.6413$ (4 d.p.) Allow awrt	A1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 = 0.05 + \cos 0.641266\ldots$ [or $0.05 + \cos(0.6 + 0.05\cos 0.6)$] $= 0.851338\ldots$	A1ft
	$y_2 \approx 0.05\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1 + y_1 \quad (= 0.05 \times 0.851338\ldots + 0.641266\ldots)$ Requires use of the differential equation to find $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_1$	M1
	$y_2 \approx 0.683833\ldots$ $= 0.6838$ (4 d.p.)	A1 **(6)**

	Degree mode in calculator: Gives answers: 0.6500 (0.64999…) 0.7025 (0.70248…) This can score B1 M1 A0 A1ft M1 A0	

Question Number	Scheme	Marks
2. (a)	$\begin{pmatrix}1 & p & 2\\0 & 3 & q\\2 & p & 1\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix}=\lambda\begin{pmatrix}1\\2\\1\end{pmatrix}$ $\quad\begin{pmatrix}1+2p+2\\6+q\\2+2p+1\end{pmatrix}=\begin{pmatrix}\lambda\\2\lambda\\\lambda\end{pmatrix}$ is M1 A1 (2 eqns implied) $\begin{pmatrix}3+2p\\6+q\\3+2p\end{pmatrix}\Rightarrow 6+q=2(3+2p)$ is M1 A1 (2 eqns, use of parameter implied) $1+2p+2=\lambda$ $\quad 6+q=2\lambda$ $\quad$ M: Two equations, one in p, one in q $\therefore\ 6+q=6+4p \ \Rightarrow\ q=4p$ (*)	M1 A1 A1 (3)
(b)	$\begin{vmatrix}-4 & p & 2\\0 & -2 & 4p\\2 & p & -4\end{vmatrix}=0$ or $\begin{vmatrix}1-\lambda & p & 2\\0 & 3-\lambda & 4p\\2 & p & 1-\lambda\end{vmatrix}=0$ (or with q instead of $4p$) $[-4(8-4p^2)-p(0-8p)+2(0+4)=0]$ $\quad p^2=1$ or $pq=4$ $p<0\quad p=-1\quad q=-4$ $\quad$ M: Use $q=4p$ to find value of p and of q A1: Positive values must be rejected	M1 A1 dM1 A1 (4)
(c)	$-4x-y+2z=0,\ -2y-4z=0,\ 2x-y-4z=0$ Any 2 eqns, with value of p $2x=-y=2z$ $\quad$ (or 2 separate equations) E.vector is $k\begin{pmatrix}1\\-2\\1\end{pmatrix}$ $\quad$ (Any non-zero value of k)	M1 M1 A1 (3) **(10)**
	(a) Assuming a value for λ, e.g. $\lambda=1$, gives M1 A0 A0. (a) Assuming result and working 'backwards': $\begin{pmatrix}1 & p & 2\\0 & 3 & 4p\\2 & p & 1\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix}=\begin{pmatrix}3+2p\\6+4p\\3+2p\end{pmatrix}=(3+2p)\begin{pmatrix}1\\2\\1\end{pmatrix}$, gives M1 A0 A0 (b) <u>Alternative</u>: $\begin{pmatrix}1 & p & 2\\0 & 3 & 4p\\2 & p & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=5\begin{pmatrix}x\\y\\z\end{pmatrix}$ or $\begin{pmatrix}-4 & p & 2\\0 & -2 & 4p\\2 & p & -4\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ (or q instead of $4p$) $x+py+2z=5x,\ 3y+4pz=5y,\ 2x+py+z=5z$ $py+2z=4x$ (i), $2pz=y$ (ii), $2x+py=4z$ (iii) From (i) and (iii) $\quad py=2z$ From (ii) $\quad p^2=1$ $\quad$ (or equiv. in terms of p and/or q) $p<0,\ p=-1,\ q=-4$ $\quad$ A1: Positive values must be rejected (b) Using the eigenvector $\begin{pmatrix}1\\2\\1\end{pmatrix}$ scores <u>no marks</u> in this part.	M1 A1 dM1 A1

Question Number	Scheme	Marks
3. (a)	$\left(x^2+1\right)\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}=4y\frac{dy}{dx}+(1-2x)\frac{d^2y}{dx^2}-2\frac{dy}{dx}$	M1 A1
	$\left(x^2+1\right)\frac{d^3y}{dx^3}=\left(1-4x\right)\frac{d^2y}{dx^2}+\left(4y-2\right)\frac{dy}{dx}$ (*)	A1 (3)
(b)	$\left(\frac{d^2y}{dx^2}\right)_0=3$	B1
	$\left(\frac{d^3y}{dx^3}\right)_0=5$ Follow through: $\frac{d^3y}{dx^3}=\frac{d^2y}{dx^2}+2$	B1ft
	$y=1+x+\frac{3}{2}x^2+\frac{5}{6}x^3....$	M1 A1 (4)
(c)	$x=-0.5,\quad y\approx 1-0.5+0.375-0.104166\ldots$ $=0.77$ (2 d.p.) [awrt 0.77]	B1 (1) **(8)**
	(a) M: Use of product rule (at least once) and implicit differentiation (at least once). (b) M: Use of series expansion with values for the derivatives (can be allowed without the first term 1, and can also be allowed if final term uses 3 rather than 3!)	

4.	(a)	$\lvert(x-3)+\text{i}y\rvert = 2\lvert x+\text{i}y\rvert \Rightarrow (x-3)^2 + y^2 = 4x^2 + 4y^2$ $\therefore\ x^2 + y^2 + 2x - 3 = 0$ $(x+1)^2 + y^2 = 4$ Centre $(-1, 0)$, radius 2	M1 A1 M1 A1, A1 (5)
	(b)	 Circle, centre on *x*-axis B1 *C* (–1, 0), *r* = 2 dB1ft Follow through centre and radius, but dependent on first B1. There must be indication of their '–3', '–1' or '1' on the *x*-axis and no contradictory evidence for their radius. Straight line B1 Straight line through (–1, 0), or perp. bisector of (–3, 0) and $(0, \sqrt{3})$. B1 Straight line through point of int. of circle & –ve *y*-axis, or through $(0, -\sqrt{3})$ B1	B1 dB1 B1 B1 B1 (5)
	(c)	Shading (only) inside circle Inside correct circle and all of the correct side of the correct line… this mark is dependent on <u>all</u> the previous B marks in parts (b) and (c).	B1 dB1 (2) **(12)**
		(a) 1st M: Use $z = x + \text{i}y$, and attempt square of modulus of each side. Not squaring the 2 on the RHS would be M1 A0. 2nd M: Attempting to express in the form $(x-a)^2 + (y-b)^2 = k$, or attempting centre and radius from the form $x^2 + y^2 + 2gx + 2fy + c = 0$	

Question Number	Scheme	Marks
5. (a)	$\begin{pmatrix} k & -2 \\ 1-k & k \end{pmatrix}\begin{pmatrix} t \\ 2t \end{pmatrix} = \begin{pmatrix} t(k-4) \\ t(1+k) \end{pmatrix}$ $t(1+k) = 2t(k-4)$ $k = 9$	M1 dM1 A1 (3)
(b)	det **A** $= k^2 + 2(1-k)$ (Must be seen in part (b)) $= (k-1)^2 + 1$, which is always positive **A** is non-singular	M1 M1 A1cso (3)
(c)	$\mathbf{A}^{-1} = \dfrac{1}{k^2 - 2k + 2}\begin{pmatrix} k & 2 \\ k-1 & k \end{pmatrix}$	M1 A1 (2)
(d)	$k = 3,\quad \mathbf{A}^{-1} = \dfrac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ $\mathbf{Ap} = \mathbf{q} \Rightarrow \mathbf{p} = \mathbf{A}^{-1}\mathbf{q} \qquad \mathbf{p} = \dfrac{1}{5}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \dfrac{1}{5}\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ Alt. $\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \Rightarrow 3x - 2y = 4, -2x + 3y = -3$ B1 M1 A1 for solving two sim. eqns. in x and y to give $x = 1.2$, $y = -0.2$ (o.e.)	B1 M1 A1 (3) **(11)**
	(b) 2nd M: Alternative is to use quadratic formula on the quadratic equation, or to use the discriminant, with a comment about 'no real roots', or 'can't equal zero', or a comment about the condition for singularity. $\left(x = \dfrac{2 \pm \sqrt{4-8}}{2}\right)$ A1 Conclusion. (c) M: Need $\dfrac{1}{\text{their det A}}$, k's unchanged and attempt to change sign for either -2 (leaving as top right) or $1-k$ (leaving as bottom left). (d) M: Requires an attempt to multiply the matrices.	

Question Number	Scheme	Marks
6. (a)	$(\cos\theta + \mathrm{i}\sin\theta)^1 = \cos\theta + \mathrm{i}\sin\theta \quad \therefore$ true for $n = 1$	B1
	Assume true for $n = k$, $(\cos\theta + \mathrm{i}\sin\theta)^k = \cos k\theta + \mathrm{i}\sin k\theta$	
	$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = (\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	M1
	$= \cos k\theta\cos\theta - \sin k\theta\sin\theta + \mathrm{i}(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$	M1
	(Can be achieved either from the line above or the line below)	
	$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1
	Requires full justification of $(\cos\theta + \mathrm{i}\sin\theta)^{k+1} = \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	
	($\therefore$ true for $n = k + 1$ if true for $n = k$) $\quad \therefore$ true for $n \in \mathbb{Z}^+$ by induction	A1cso (5)
(b)	$\cos 5\theta = \mathrm{Re}\left[(\cos\theta + \mathrm{i}\sin\theta)^5\right]$	
	$= \cos^5\theta + 10\cos^3\theta\,\mathrm{i}^2\sin^2\theta + 5\cos\theta\,\mathrm{i}^4\sin^4\theta$	M1 A1
	$= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	M1
	$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$	M1
	$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*)	A1cso (5)
(c)	$\dfrac{\cos 5\theta}{\cos\theta} = 0 \quad \Rightarrow \quad \cos 5\theta = 0$	M1
	$5\theta = \dfrac{\pi}{2} \qquad \theta = \dfrac{\pi}{10}$	A1
	$x = 2\cos\theta, \qquad x = 2\cos\dfrac{\pi}{10}$ is a root (*)	A1 (3)
		(13)
	(a) Alternative: For the 2nd M mark: $(e^{\mathrm{i}k\theta})(e^{\mathrm{i}\theta}) = e^{\mathrm{i}\theta(k+1)}$ (b) Alternative: $\left(z+\frac{1}{z}\right)^5 = z^5 + 5z^4\left(\frac{1}{z}\right) + 10z^3\left(\frac{1}{z}\right)^2 + 10z^2\left(\frac{1}{z}\right)^3 + 5z\left(\frac{1}{z}\right)^4 + \left(\frac{1}{z}\right)^5$ M1 $= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$ A1 $(2\cos\theta)^5 = ...$ and attempt to put $\cos 3\theta$ in powers of $\cos\theta$ M1 Correct method (or formula) for putting $\cos 3\theta$ in powers of $\cos\theta$ M1 $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ A1cso (c) Alternatives: (i) Substitute given root into $x^4 - 5x^2 + 5$: $\left(2\cos\frac{\pi}{10}\right)^4 - 5\left(2\cos\frac{\pi}{10}\right)^2 + 5 = 2^4\left(\cos\frac{\pi}{10}\right)^4 - 5\times 2^2\left(\cos\frac{\pi}{10}\right)^2 + 5$ M1 'Multiply by $\cos\theta$' and use result from part (b): $... = \cos\frac{5\pi}{10}$ A1 $= 0$ and conclusion A1 (ii) Use $5\theta = \frac{\pi}{2}$ in result from part (b) M1 $16\left(\cos\frac{\pi}{10}\right)^5 - 20\left(\cos\frac{\pi}{10}\right)^3 + 5\left(\cos\frac{\pi}{10}\right)$ and divide by $\cos\theta$ A1 $= 0$ and conclusion A1	

Question Number	Scheme	Marks
7. (a)	$\overrightarrow{PQ} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\overrightarrow{PR} = 2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1
	$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$	M1 A1 (3)
(b)	$\mathbf{r} \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = (\mathbf{i} - \mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k})$ [may use $\overrightarrow{OQ}$ or $\overrightarrow{OR}$]	M1
	$\mathbf{r} \cdot (3\mathbf{i} + \mathbf{j} - \mathbf{k}) = 4$ o.e. ft from (a)	A1ft (2)
(c)	$3x + y - z = 4$ (i), $x - 2y - 5z = 6$ (ii)	
	(i) × 2 + (ii) $7x - 7z = 14$, $x = z + 2$ (M: Eliminate one variable)	M1
	In (ii) $z + 2 - 2y - 5z = 6$, $y + 2 = -2z$ (M: Substitute back)	M1
	$\therefore x = z + 2$ and $y + 2 = -2z$ o.e. $(y = 2 - 2x)$ (Two correct '3-term' equations)	A1
	$\dfrac{x-2}{(1)} = \dfrac{y+2}{-2} = \dfrac{z}{(1)}$ o.e. (M: Form cartesian equations)	M1 A1 (5)
(d)	Writing down direction vector of $\overrightarrow{PS}$ from part (c).	M1
	$\overrightarrow{QR} = \mathbf{i} - 2\mathbf{j} + \mathbf{k} = \overrightarrow{PS}$ $\therefore PS$ // QR (or cross-product = 0)	A1 (2)
(e)	$\overrightarrow{PT} = 4\mathbf{i} + 2\mathbf{j}$ (or $\overrightarrow{QT} = 3\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ or $\overrightarrow{RT} = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$)	
	Volume = $\frac{1}{3}\left\|\overrightarrow{PQ} \times \overrightarrow{PR} . \overrightarrow{PT}\right\| = \frac{1}{3}\left\|(3\mathbf{i} + \mathbf{j} - \mathbf{k}) \cdot (4\mathbf{i} + 2\mathbf{j})\right\|$ ft from (a) (Instead of $\overrightarrow{PQ} \times \overrightarrow{PR}$, it could be $\overrightarrow{PQ} \times \overrightarrow{QR}$ or $\overrightarrow{PR} \times \overrightarrow{QR}$)	M1 A1ft
	$= \frac{1}{3}(12 + 2)$	
	$= 4\frac{2}{3}$ o.e.	A1 (3)
		(15)
	(a) If both vectors are 'reversed', B0 M1 A1 is possible (c) <u>Alternative</u>: Direction of line: $\begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = 7\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ M2 A1 Through P $(1, 0, -1)$: $\dfrac{x-1}{1} = \dfrac{y}{-2} = \dfrac{z+1}{1}$ M1 A1 (e) <u>Alternative</u>: $\frac{1}{3}\begin{vmatrix} 4 & 2 & 0 \\ 1 & -1 & 2 \\ 2 & -3 & 3 \end{vmatrix}$ gives M1 A1 directly. Here ft from 1st line of part (a). <u>Special case</u>: $\frac{1}{6}$ or $\frac{1}{2}$ instead of $\frac{1}{3}$, but method otherwise correct: M1 A0 A0	